JOURNAL OF SPACECRAFT AND ROCKETS
Vol. 35, No. 5, September–October 1998

Spacecraft Momentum Dumping Using Gravity Gradient

Donald Tong*
The Boeing Company, Seattle, Washington 98124-2499

The disturbance angular momentum accumulated by the momentum wheels or control momentum gyros onboard an orbiting spacecraft can be alleviated with gravity-gradient torque in some cases. How a modification to the nominal spacecraft operational attitude profile can result in lowering propellant consumption, potentially extending the spacecraft's life span on orbit, is discussed. The feasibility of using gravity gradient for momentum dumping is demonstrated in a case study.

Introduction

THE disturbance experienced by a spacecraft on orbit consists of solar radiation pressure, aerodynamic drag, gravity-gradient (GG) torque, magnetic torque, etc. For a three-axis stabilized spacecraft, this disturbance torque is counteracted by an attitude control device, such as momentum wheels or control momentum gyros. The accumulated angular momentum is stored (conservation of angular momentum), and when the device is saturated, it is usually unloaded by firing the reaction jets or using magnetic torquers. The penalty of using reaction jets includes fuel consumption, spacecraft disturbance, and contamination. In addition, thruster impulse causes uncertainties in estimating vehicle orbit parameters, and this is undesirable if spacecraft ephemeris knowledge is an important performance parameter. Magnetic torquers are generally costly and have low control authority for large momentum dumping demand.

Over the past 25 years, studies of momentum control using GG torque of several spacecraft (Skylab,[1] space station,[2–4] etc.) have been published. For these systems, the penalty associated with control moment gyro (CMG) desaturation using reaction jets was a main concern. Therefore, the use of GG torque was considered early in the attitude control design. One example of a control system design for a very specific orbit and vehicle configuration is illustrated by Powell.[5]

For most smaller spacecraft, however, GG is considered as a disturbance torque and not harnessed for CMG desaturation due to the conflict of GG attitude with spacecraft nadir pointing requirements for communication, Earth observation, broadcasting, etc. The penalty associated with desaturation has just been accepted.

It is suggested that, in certain cases, a simple GG attitude command profile can be sent to the spacecraft without affecting the intended functions of the original profiles. One such case, an asymmetrically configured spacecraft with a single axis gimbaled and single panel solar array, flying in a nadir pointing attitude profile in an elliptical orbit, is used as an example to show the benefit of the approach.

Vehicle Disturbance Torque

GG

Whether in a low Earth orbit or in an elliptical orbit, when any part of an spacecraft orbit is sufficiently low near the Earth (altitude below ~4000 km), and if the spacecraft is not at a stable attitude, gravity acceleration differences in the gravitational field (GG) acting on the various mass elements of the vehicle cause a disturbance torque of significant magnitude about the center of mass. The spacecraft is at its stable attitude position in the gravity field when the vehicle minor principal axis (minimum moment of inertia axis) is perpendicular to the gravitational equipotential. For an asymmetrically configured spacecraft (principal moment of inertia of the orthogonal axes having different values), GG torque may be significant near perigee if the vehicle z principal axis deviates from the nadir vector. This can happen even if the body z axis is kept pointing at the Earth nadir because the positions of the spacecraft appendages can cause the principal axes to move away from the body axes.

Solar Pressure

Solar radiation pressure is usually an unwanted disturbance and a significant one for spacecraft employing only a single solar panel without a solar sail for counterbalance. For an asymmetrically configured spacecraft (Fig. 1) flying in an elliptical and with a sun-nadir attitude profile (bus z axis to nadir and solar array to sun), solar radiation pressure can build up significantly and saturate the momentum control device within a few orbit revolutions. In an elliptical orbit, the spacecraft spends more time on the apogee side, i.e., $90 <$ true anomaly < 270 deg. Therefore, a single solar panel and sun-nadir following spacecraft has a net angular momentum gain after a complete revolution. Because of orbit symmetry, spacecraft that follow circular orbits may have only small angular momentum accumulation. Earth eclipsing of the spacecraft also affects the accumulation of solar radiation pressure angular momentum.

Aerodynamic

Aerodynamic forces also create a disturbance torque if the spacecraft's orbital perigee is low. Aerodynamic torque becomes significant below the altitude of 1000 km.

Magnetic Disturbance Torque

A net magnetic disturbance torque exists due to the interaction between the Earth's magnetic field and the magnetism generated by the various spacecraft components and devices. The value of this disturbance angular momentum is relatively small and is difficult to estimate. The magnetic disturbance torque is not considered in this study.

Spacecraft Operation Constraints

When developing the command attitude profile for momentum dumping with GG torque, certain operations constraints have to be satisfied.

1) The solar array normal vector has to be within the off-sun angle limit (usually 10–20 deg).

2) Vehicle rate and acceleration capabilities of the control system may not be exceeded (usually 0.05–0.5 deg/s and 0.005–0.05 deg/s^2).

3) Thermal requirements for the spacecraft subsystems have to be satisfied, e.g., keeping radiators and temperature-sensitive subsystems away from the sun. (The vehicle usually has a hot side and a cold side.)

4) Functionality and intentions of the original bus attitude profiles have to be maintained (antenna gimbal/coverage capability for Earth communications, observation, etc.).

5) Other mission-specific requirements.

GG Momentum Dumping Case Study

A computer program was developed to study the feasibility for different spacecraft configurations and sun–orbit geometry by

Received July 22, 1997; revision received April 30, 1998; accepted for publication May 9, 1998.

*Principal Engineer, Boeing Information, Space and Defense Systems, M/S 8F-05, P.O. Box 3999.

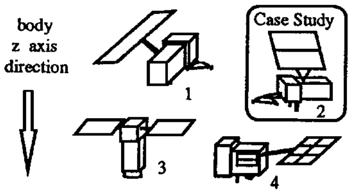

Fig. 1 Examples of asymmetrically configured spacecraft.

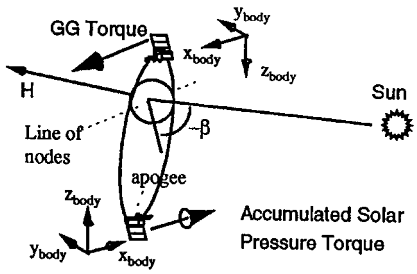

Fig. 2 GG torque for momentum dumping.

generating GG attitude profiles (or GG maneuver) for momentum dumping. The study involves the hypothetical CMG controlled spacecraft (vehicle 2 shown in Fig. 1) with a single axis gimbaled and single panel solar array flying with a sun-nadir profile in an elliptical orbit[6] (perigee altitude ~1000 km). The most dominant disturbance angular momentum in this orbit is due to GG and solar radiation. The GG torque near perigee will be harnessed (by commanding the vehicle attitude) for dumping the accumulated momentum. The net solar pressure disturbance momentum tends to accumulate in the inertial direction, the same as the $+x$ body axis direction when the vehicle is at apogee ($-x$ body axis direction at perigee). This is due to the spacecraft spending a large percentage of time on the apogee side of the line of nodes in this particular orbit. Figure 2 shows the orbit, body coordinates, and momentum vectors.

Under nominal operations in orbit, the spacecraft follows the sun-nadir attitude profiles to provide continuous solar power and ground communications/observations. The vehicle z axis points to the Earth nadir to satisfy the communication and observation requirements. The x axis is maintained in the plane perpendicular to the vehicle-to-sun vector such that the solar array can remain exposed to the sun by rotating the solar panel. The y axis is the vector normal of the cold spacecraft surface, and the $-y$ axis is the normal of the hot side. The vehicle is required to execute a z-axis yaw turn in the seasons when the sun is near the orbit plane ($\beta \sim 0$ in Fig. 2) close to the line of nodes, to satisfy the sun-nadir attitude operation.

There are no data communication and observation requirements when the spacecraft is near perigee, out of view from the ground station. However, the spacecraft is still required to complete the perigee turn operating below the vehicle rate and acceleration limits.

Momentum Dumping Using GG

The GG torque equation[7] is

$$\text{torque}_{\text{GG}} = (3\mu/R^3)[\hat{\boldsymbol{R}} \times (\boldsymbol{I} \cdot \hat{\boldsymbol{R}})] \tag{1}$$

where

μ = Earth's gravitational constant (3.986E5 km^3/s^2)
$\hat{\boldsymbol{R}}$ = geocenter unit vector in body coordinates
$\boldsymbol{I}$ = spacecraft moment of inertia tensor

Because there is no nadir pointing requirement near perigee, GG torque can be utilized to help offset the accumulated angular momentum. The vehicle attitude profile near perigee will have to deviate from the nominal sun-nadir profile, i.e., $\hat{\boldsymbol{R}} \neq [0, 0, -1]$. Notice that the magnitude of the torque is inversely proportional to R^3.

It is helpful to use small angle approximation to visualize the relationship between the vehicle attitude, mass characteristics, and the GG torque. Based on Eq. (1) and assuming small angle deviation[8] from the nominal spacecraft attitude, where z is parallel to the nadir vector,

$$\hat{\boldsymbol{R}} \cong \theta \cdot \hat{\boldsymbol{i}} - \phi \cdot \hat{\boldsymbol{j}} - \hat{\boldsymbol{k}}$$

$$\boldsymbol{I} \cdot \hat{\boldsymbol{R}} \cong I_x \cdot \theta \cdot \hat{\boldsymbol{i}} - I_y \cdot \phi \cdot \hat{\boldsymbol{j}} - I_z \cdot \hat{\boldsymbol{k}}$$

$$\text{torque}_{\text{GG}} \propto \hat{\boldsymbol{R}} \times (\bar{\boldsymbol{I}} \cdot \hat{\boldsymbol{R}})$$

$$\propto (I_z - I_y) \cdot \phi \cdot \hat{\boldsymbol{i}} + (I_z - I_x) \cdot \theta \cdot \hat{\boldsymbol{j}} + (I_x - I_y) \cdot \theta \cdot \phi \cdot \hat{\boldsymbol{k}}$$

where

ϕ = small principal axis roll from nadir pointing
θ = small principal axis pitch from nadir pointing

Notice body yaw angle does not apply because it does not affect the unit vector $\hat{\boldsymbol{R}}$. The following observations are made by examining the resulting equation, which is proportional to the GG torque. The GG $z(\boldsymbol{k})$ component torque is negligible because $\theta \cdot \phi$ is small. The $y(\boldsymbol{j})$ component of the GG torque is not beneficial because it would be perpendicular to the accumulated solar pressure momentum vector. Therefore, the vehicle pitch attitude is not commanded to deviate from the nominal nadir pointing profile, $\theta = 0$. GG torque in the $x(\boldsymbol{i})$ direction is desirable for momentum dumping, as shown in Fig. 2. The term $(I_z - I_y) \cdot \phi \cdot \hat{\boldsymbol{i}}$ determines the magnitude of the usable GG torque. For this study spacecraft, there is a significant difference of values in I_y and I_z. (With the solar panel in null position, $I_y = 40{,}000$ and $I_z = 24{,}000$ kg-m^2 or $I_y = 29{,}500$ and $I_z = 17{,}700$ slug-ft^2.)

Setting the pitch and yaw angles to zero based on the results of the preceding small angle analysis, Eq. (1) can then be simply carried out as

$$\hat{\boldsymbol{R}} = [0, \sin(\text{roll}), \cos(\text{roll})]$$

$$\therefore \text{torque}_{\text{GG}} \propto (I_z - I_y) \times \sin(\text{roll}) \times \cos(\text{roll})$$

It then can be concluded that, for constant moment of inertia values (fixed appendage positions), maximum torque is achieved at the principal axis (not necessarily the body z axis) roll angle of 45 deg from nadir. In reality the solar array panel gimbal position varies (causing the values of inertia to change) as the bus maneuvers, suggesting a varying principal axis roll angle at each time step is optimal. However, it was elected to use a constant roll angle for each perigee pass. A constant principal axis roll angle results in less erratic attitude and solar array profiles. The solar array gimbal travel is limited to keep the value of $(I_z - I_y)$ large. As functions of sun-orbit geometry, the roll angle magnitude and gimbal limits were obtained by an empirical method, running the computer program optimizing fuel consumption. The optimized roll angle varies from 30 to 45. Gimbal limits were determined to be -35 and $+60$ deg. These optimized parameters are then used by the computer program for future runs with various sun-orbit geometry.

It appears that the best solution is a negative vehicle roll from the original sun-nadir attitude. Because the bus z axis relative to the z principal axis varies with solar array position, which changes as the bus is maneuvering, the bus GG roll attitude is obtained at every time step by iteration. When the vehicle is at the properly computed roll angle passing through perigee, a GG torque is generated in the inertial direction opposite to the direction of the undesirable accumulated disturbance momentum. Figure 3 shows this spacecraft attitude for momentum dumping utilizing GG. The solar array panel is skewed off sun to help to achieve the z principal axis offset but is kept within the off-sun angle limit. The vehicle z axis is operationally free to move away from the Earth nadir in this period as long as the solar array panel off-sun angle is within limit and thermal constraints are satisfied. In this case study, momentum dumping using GG is a feasible method.

Because of the off-sun angle constraints and limited vehicle body rate control capabilities, the desired principal z-axis body roll angle is not always achievable. In this case, the vehicle is rolled as far as possible while satisfying the operation constraints.

To help visualize the vehicle attitude near perigee, the bus rates were plotted. Figures 4 and 5 show the vehicle rates for a nominal

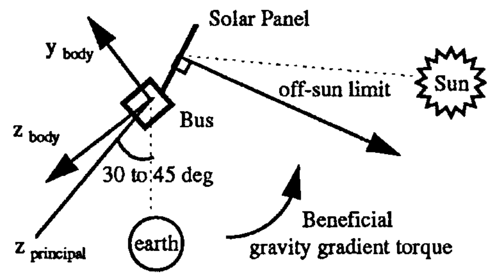

Fig. 3 Spacecraft attitude for momentum dumping using GG near perigee.

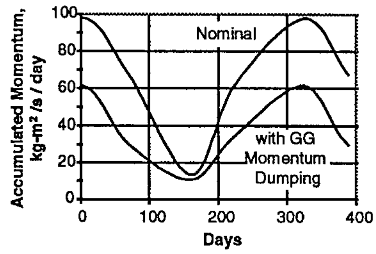

Fig. 8 Accumulated angular momentum with and without GG maneuver.

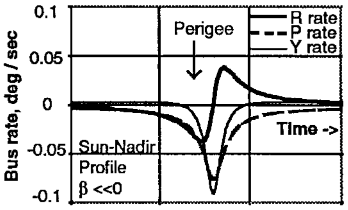

Fig. 4 Bus rates for a sun-nadir attitude profile in a negative β sun-orbit season.

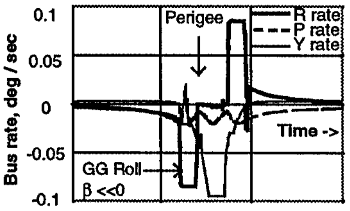

Fig. 5 Bus rates for the GG attitude profile in a negative β sun-orbit season.

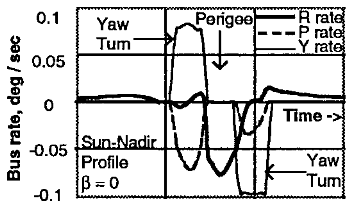

Fig. 6 Bus rates for a sun-nadir attitude profile when sun is in orbit plane.

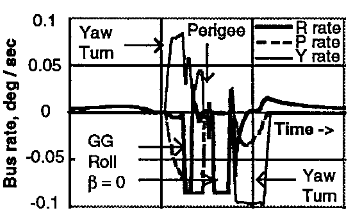

Fig. 7 Bus rates for the GG attitude profile when sun is in orbit plane.

sun-nadir profile and the GG profile, respectively, in a negative β season. (The sun is in the opposite direction of the orbit angular momentum H.) Here β is the sun angle measured from the sun vector projection on the orbit plane, as shown in Fig. 2. Bus rates are relatively small near apogee and are not shown in these plots.

In certain seasons the yaw turn maneuvers may well extend into the orbit path near perigee, where the GG maneuver needs to be. In these yaw turn seasons when β is small, the GG vehicle roll maneuver is combined with the yaw turns. Figures 6 and 7 show the vehicle rates near perigee with and without the GG maneuvers, respectively, for $\beta \sim 0$ (sun vector in the orbit plane).

When the vehicle is near either the descending node or ascending node away from perigee, it experiences a much lower level of GG torque due to the greater distance from the Earth. In addition, the GG output axis (vehicle x) is not at the desirable direction because either the vehicle is in the middle of the yaw turn or the x axis is perpendicular to the desired direction. Therefore, the effective period of performing momentum dumping with GG is between the descending and the ascending nodes centered about perigee.

CMG Desaturation Fuel Consumption Savings

An interesting question now is how much fuel can be saved with the GG maneuver in this scenario. The computer program assumes the amount of fuel used for CMG desaturation is directly proportional to the amount of disturbance angular momentum accumulated by the CMG. This is a good assumption if the CMG desaturation operations using thrusters are performed in the most efficient way: Desaturation torque axis is aligned directly opposite to the accumulated momentum vector. When the spacecraft is commanded to follow the GG attitude profile, the value of the accumulated angular momentum is reduced due to the beneficial contribution of the GG torque associated with the GG attitude profile. This magnitude difference of the nominal accumulated momentum and the accumulated momentum with the GG maneuver over a multiple of orbit revolutions is used to estimate the percentage of fuel savings.

Figure 8 shows the nominal (with sun-nadir attitude profile) accumulated disturbance angular momentum per day that the CMGs have to store. The momentum variation shown in Fig. 8 reflects the changing sun-orbit geometry due to yearly Earth revolution around the sun and the effect of orbit nodal regression. The total nominal angular momentum accumulated in a year (area under the curve) is 23,680 kg-m^2/s (17,466 ft-lb-s). Figure 8 also shows the accumulated momentum history if the spacecraft is commanded to follow the GG attitude profile around perigee. The angular momentum accumulation for the spacecraft flying with GG maneuver is only 13,380 kg-m^2/s per year (9870 ft-lb-s), equivalent to more than 40% fuel savings over the nominal case. The GG momentum dumping maneuver is assumed to be performed every orbit revolution.

One of the most significant factors of GG torque effectiveness is the value of the spacecraft radius R from the center of the Earth. The GG torque is inversely proportional to R^3. Long-term orbit parameter prediction suggests that perigee altitude of the orbit increases from a minimum of 1000 km to a maximum of 2000 km. The level of GG torque will decrease nearly 30% when the vehicle's perigee altitude is at its maximum. The perigee altitude variation has a period of several years and is caused by the combined gravity pull of the sun and the moon. Therefore, the perigee altitude variation in the life of the vehicle should be considered when estimating the long-term effectiveness of the GG maneuver.

Summary

A computer program has been developed to generate a GG momentum dumping attitude profile and CMG desaturation fuel savings estimates. An innovative algorithm is used to allow the tool to generate the proper GG roll and solar array profiles for various sun-orbit geometries and overlay it on the existing nominal sun-nadir profiles. The impact on the original operational design is then kept at a minimum. Fuel savings estimates are computed based on the accumulated angular momentum difference between the nominal case and the case with the GG maneuver.

A case study has shown that, for a certain asymmetrically configured spacecraft flying in an elliptical orbit, nominal spacecraft attitude near perigee can be modified to dump a significant amount of the accumulated disturbance momentum using GG torque. The optimal GG maneuver to counter the accumulated angular momentum was found to be a vehicle roll (about the body negative x axis) moving the vehicle principal z axis (not necessarily the body z axis) 30–45 deg from nadir.

Although complete momentum dumping is not achieved for the vehicle in the case study, the analysis has demonstrated that a significant attitude maintenance CMG desaturation fuel saving (over

40%) is attainable, potentially extending the spacecraft's life span. For spacecraft operating in circular orbits, the disturbance momentum accumulation due to solar pressure is less significant, and the results of the study may not apply.

Acknowledgments

The author expresses his appreciation to F. Brontsema, E. Estabrook, and N. Hughes among others for advice in performing this analysis.

References

[1] Ross, C. H., and Worley, E., "Optimized Momentum and Attitude Control System (MACS) for Skylab," AIAA Paper 71-938, Aug. 1971.

[2] Woo, H. H., Morgan, H. D., and Falangas, E. T., "Momentum Management and Attitude Control Design for a Space Station," *Journal of Guidance, Control, and Dynamics*, Vol. 11, No. 1, 1988, pp. 19–25.

[3] Wie, B., Byun, K. W., Warren, V. W., Geller, D., Long, D., and Sunkel, J., "New Approach to Attitude/Momentum Control for the Space Station," *Journal of Guidance, Control, and Dynamics*, Vol. 12, No. 5, 1989, pp. 714–722; also AIAA Paper 88-4132, Aug. 1988.

[4] Harduvel, J. T., "Continuous Momentum Management of Earth-Oriented Spacecraft," *Journal of Guidance, Control, and Dynamics*, Vol. 15, No. 6, 1992, pp. 1417–1426.

[5] Powell, B. K., "Gravity Gradient Desaturation of a Momentum Exchange Attitude Control System," AIAA Paper 71-940, Aug. 1971.

[6] Chobotov, V. A., *Orbital Mechanics Control*, AIAA Education Series, AIAA, Washington, DC, 1991, pp. 288–296.

[7] Wertz, J. R., *Spacecraft Attitude Determination and Control*, D. Reidel, London, 1986, pp. 566–568.

[8] Bryson, A. E., Jr., *Control of Spacecraft and Aircraft*, Hemisphere, Washington, DC, 1975, pp. 7, 8.

A. C. Tribble
Associate Editor